SCIENCE

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*THE GROUNDWORK OF DYNAMICS.**

THE subject of dynamics is too often treated as if its chief value consisted in the opportunities it affords for familiarizing the student with the operations of the differential and integral calculus. It is regarded as a department of applied mathematics rather than of mechanical science. That this should be the case is unfortunate; at the same it is not in the nature of things altogether avoidable. The student cannot afford the time involved in deferring the study of dynamics until he has acquired a working knowledge of the calculus. As a consequence he becomes confused respecting the origin of his difficulties, and possibly attributes to his ignorance of mathematics misconceptions the nature of which may be purely dynamical. It would be of great benefit to him to have the opportunity of attending, before the close of his studies, a short course of lectures on the fundamental principles of the subject, that is to say, the conceptions springing directly from experience, upon which the science is founded. The mind of the individual resembles in its mode of growth the mind of the race. The study of the historical development of mechanical ideas will go a great way in clearing up difficulties which arise from adhering too closely to one line of thought. Many of the greatest advances in science are the result of changes of the point of view. Such changes correspond in some measure with the process known to the mathematician as the transformation of coordinates, a process which often transforms a mass of brain-wearying symbols into ordered groups, instinct with life and

* Address by the Vice-President before Section D. of the Detroit Meeting of the American Association for the Advancement of Science.

meaning. It will be well, therefore, for the student of dynamics occasionally to make the endeavor to transform his coordinates and view the subject from different standpoints. For this purpose it is unnecessary that he should be an expert in refined mathematical analysis. It is requisite, on the other hand, that he should possess, in some degree, what may be called the mechanical instinct. The whole power of modern analysis has not proved sufficient to solve, in its generality, the problem of the three bodies; a problem extremely useful, nevertheless, as an illustration of dynamical principles.

The science of statics, or the laws of composition of forces and couples, was developed from rude experiments made with springs and with strings and weights; the tensions of the strings being measured by the weights. The conditions necessary in these experiments were that the body to which the springs or strings were fastened should be maintained by them at rest, and that all changes of shape and size should have ceased. Statics was thus established without the introduction of the ideas of mass and acceleration. In this stage, however, its laws were supposed to apply only to rigid bodies at rest. The facts to be noted are, that force was recognized as a fundamental conception, and that methods of measuring it and the laws of composition of forces were discovered, without reference to motion except in the respect that rest was supposed to be a necessary condition.

The connections between force and motion, the subject-matter of dynamics, were established by observing the motions of falling bodies, pendulums, bodies rolling and sliding down inclined planes, colliding spheres, bodies connected by strings running over pulleys, etc. In making these experiments it was necessary to know the forces acting. As the only scientific knowledge of force at the time was contained in the laws of statics, the assumption was made that these laws were true, even though the bodies were moving and whether the velocities were increasing, decreasing, or changing in direction. This was a change of the point of view which was fruitful in important results. It was found, however, that while the above assumption was justified in the case of the composition of forces, and in the case of weights when considered as forces acting on the heavy bodies themselves, it was not true to assume that the tension of a string in motion is measured by the attached weight. The true indication of the tension in a string was recognized to be the same as in the case of a spring, viz., the elongation. Again, while in statics, the principal objects were the strings and weights, and the bodies to which they were attached were of little or no account, in the dynamical experiments the bodies assumed importance. The conception of mass was introduced, and was found to correspond with the commercial idea of quantity of material, as determined by the balance and weights. The results of these experiments are contained in the laws of dynamics, or the laws of motion, as they are usually called, which may be summarized as follows:

I. That no change in a body's motion of translation takes place except by the action of external forces.

II. That external forces impress on a body changes of momentum in their own directions at rates proportional to their magnitudes.

III. That action and reaction are equal and opposite, and in the same line.

It seems to be a matter of doubt whether Newton, in his statement of the laws of motion which I have thus paraphrased, intended to affirm that action and reaction are in the same line. Whether there be ground for this doubt or not, the idea is

implied in D'Alembert's principle and is accepted as true.

The third law is a law of force, the value of which is seen when the mutual actions of bodies are to be considered. Without this law the laws of statics and of motion would refer to actions on one body only.

It was evident that the laws of composition of forces could be considered as corollaries of the laws of motion, so that from the latter a more comprehensive view of the measurement of force was gained than from the statical experiments. Thus the statical method of measuring force by balancing the tension of one string or spring against that of another was interpreted dynamically, by assuming that the body upon which the springs acted remained at rest in consequence of receiving from the forces opposite changes of momentum at the same rate. It followed that if the springs were to act singly they would, other things being equal, produce equal rates of change of momentum. Thus dynamics furnished a new method of measuring forces which agreed with that by means of springs.

Further investigation in the light of the new principles showed that the method of measuring forces by weights agreed with the spring measurement and with the dynamical measurement when the condition was observed that the weights were kept in the same locality, but in general not otherwise. Dynamics gave an explanation of this anomaly by showing that the forces of the weights were due to gravity, which evidently might change in intensity from place to place, although the masses of the weights remained unaltered. The measurement of force by weights was thus shown to be a special case of the general dynamical method. Also it resulted that the measurement of force by weights was in principle the same as the measurement by springs, the weight and the earth together constituting a spring whose elastic force is gravity.

It is of little use in the early dynamical training of a student to dwell on the fact that the unit of force in dynamical measure is a force which produces unit acceleration in unit mass. It may even do harm. It may have the effect of leading him to believe that the refined methods of measuring force by means of weights and springs, which are the only methods used in the laboratory, are wrong and that the testing machines and pressure gauges of the engineer are beneath contempt. It ought to be impressed upon him that all the methods of measuring force approved by experience are equally scientific and equally absolute, and will give exactly the same results, aside from unavoidable experimental error, provided that the proper conditions in each case are observed; also that the choice of the method depends, just as in the case of everything else in life, upon the objects in view and the circumstances.

We now come to the period of the development and extension of the laws of motion. While proved in the case of small bodies and motions of limited range, it by no means followed that they applied to the motions of the tides or of the planets. Newton, by the aid of his happy intuition, the law of gravitation, put new meaning into them and extended their jurisdiction over all visible and measurable motions within the solar system.

All motion is, so we are taught, relative; and the motion which is uniform and in a straight line with reference to one set of axes may be varying and in a very crooked line when referred to another set, and perfect rest with regard to a third set. The question then is, for what axes are the laws of motion true? It is very certain that the stresses in the springs by which forces are measured are in no wise affected by the choice of axes of reference.

Imagine a cube of rubber, with several points marked on its faces, and consider the lines joining one of these points with the others. The angles between these lines measure the relative directions of the terminal points from the initial point. Now set the cube in motion ; stresses and strains are generated, in consequence of which the relative directions of the lines undergo small alterations. Why do these changes of direction remain within small limits? Evidently on account of the nature of the material connecting the points.

Consider now another system of points whose relative directions as time goes on remain almost unchanged, viz., the fixed stars. Is it possible that they may be moving like the points on the rubber cube? Observational astronomy indicates that they are at immense distances from the earth and from each other. If the law of gravitation holds for them their mutual attractions must be so feeble that they form a practically unconnected system. The constancy of their relative directions cannot therefore be accounted for as in the former case by the action of the matter between them, but must be attributable to some other cause. The distinction between the two cases, which is very real, is recognized by assuming that a line may have absolute direction; and the stars are said to preserve their absolute directions from the earth. Again, the assumption that both the law of gravitation and the laws of motion are true for the solar system leads to the result (in consequence of its vast distance from the stars) that its center of mass has little or no acceleration; in other words, is either at rest or in uniform motion in a straight line.

Now, although the ideas of absolute direction and absolute rest or uniform motion in a straight line may from the kinematical point of view be incomprehensible, yet in dynamics these terms indicate very definite facts. It is only by the choice of the center of mass of the solar system as origin and by using the stars to fix the directions of the axes that it is possible to make the observed motions of the planets fit, at the same time, the law of gravitation and the laws of motion. It is unnecessary to enter into the details of the work; suffice it to say, it is a process of trial and error, of assumption, computation and check by observation.

The following dynamical consideration enables, in certain cases, a more convenient origin to be used than the center of mass of the solar system.

If equal accelerations are impressed on the bodies whose motions and mutual forces are under consideration, by the attraction of the remainder of the solar system, it is evident that their motions referred to an origin at their common center of mass, with axes fixed in direction by the stars, will be affected only by their mutual actions. Also assuming the laws of motion to apply, these motions so measured will show the whole effect of their mutual actions, since the latter have no effect on the motion of their center of mass. It is thus, possible, in discussing the lunar tides, to use as origin the center of mass of the earth and moon, and in the case of terrestrial bodies the center of mass of the earth. It may be noted that whether the primary origin or such subsidiary origins be used, the equations for the mutual forces under consideration are the same; the effect of the change of origin appearing only in the constants of integration. It will be convenient to term all such sets of axes, including the primary set, absolute axes.

It now remains to determine whether the absolute axes will give results agreeing with those already obtained in the small scale experiments, with axes fixed in the earth; and if they do not, whether the discrepancies can be explained.

If no explanation could be given of such discrepancies it is evident that the science of dynamics would be resolved into a bundle of empirical rules, describing the various axes of reference that applied in different cases, and the range of applicability in each case.

In order to make the comparison, it will be necessary to obtain the data required for transformation from the absolute set of axes with origin at the earth's center, to axes fixed in the earth with origin in the locality of the experiments. These data are furnished by astronomical observations. When the transformation is made there appear on the left-hand side, let us say, of the equations the rate of change of momentum of the body relative to the axes fixed in the earth, and on the right-hand side the attractions, tensions and other impressed forces, together with certain terms involving the relative motion of the two sets of axes.

In the equations of the original experiments no terms of the latter kind occurred. There are three ways of accounting for the difference. Either the forces are different in the two sets of equations, or the new terms are so small as to be within the limit of experimental error, or each experiment or class of experiments requires its special set of axes.

Experience shows that the explanation lies in the first or second alternative; the the third is not true.

These terms are generally negligible in laboratory experiments. It is necessary to consider them in the theory of winds and ocean currents. Their presence in the equations has suggested certain experiments with pendulums and gyrostats, which confirm their truth. We are justified by experience, for instance, in believing that in the northern hemisphere moving bodies tend to the right, in the southern hemisphere to the left; bodies moving eastward tend to rise, westward to fall; and that bodies, whether at rest or in motion, tend to move outwards from the polar axis. All such tendencies are represented by the terms under consideration. They may be regarded when written on the force side of the equations as representing relative or fictitious forces; fictitious because they correspond to no actions of matter, but are the consequence simply of the motion of the axes of reference relative to the absolute axes.

Sometimes it happens, as has been indicated, that the discrepancy lies in the fact that the forces in the two sets of equations are different, although referring to the same experiment. Consider the case of a body suspended from a spring. Referred to axes fixed in the earth it is at rest, and the inference is that the attraction of the earth is equal and opposite to the tension of the spring. Referring, however, to the same axes by transformation from the absolute axes, there appears, in addition to the terms representing the tension of the spring and the attraction of the earth, a new term, a relative or fictitious force, known as the centrifugal force. The inference now is that the attraction of the earth is greater than the tension of the spring, instead of being equal to it. If this inference be accepted as the true one the question arises, which of the original forces was wrong, or were both astray? Remembering that the intrinsic indication of the force exerted by the spring is its elongation, and that of attraction the acceleration caused by it, also that acceleration depends on the choice of axes of reference, while the elongation of a spring does not, there can be no hesitation in deciding that the error lay altogether in the estimate of the attraction. The fictitious force, while itself invisible, also rendered invisible a portion of the earth's attraction. By using proper axes of reference its true character is revealed and

its power for evil destroyed. If the mechanism of attraction were not concealed, or if it had some distinguishing mark other than acceleration, and it were possible to experiment with it as with springs, such an error could not be made, even with improper axes of reference.

A convenient way of regarding the laws of motion is to consider, as before, the second law as affirming the relation between force and change of momentum, and the third and first as asserting the principle of conservation of momentum; the third implying that momentum passes from one body to another without change, and the first that the only way by which the momentum of a body can suffer alteration is by part of it passing into another body. Again, if it be assumed that the third law implies that action and reaction are in the same straight line the principle of the conservation of angular momentum will follow.

The statement is sometimes made that the 'bodies' of Newton's laws must be regarded as particles. I cannot take this view; they are real bodies, of all sizes, and with all the qualities known and unknown of such bodies. They are not the imaginary bodies of the mathematician, the dramatis personæ of the algebraic theatre, possessing only the qualities arbitrarily assigned to them for the special purpose of the investigation in hand.

The laws of dynamics thus hold for all bodies within the solar system whose masses, forces and motions have hitherto been observed and measured; but the motions must be measured with essential reference to only one set of axes, namely, a set whose origin is in the sun and whose directions are fixed by the stars.

Kinematics deals with relative motion; Dynamics with the 'Motus Absolutus' of the Principia.

We now pass to the consideration of the laws of energy in their dynamical relations.

In the discussion of statics as the forerunner of dynamics, attention was directed mainly to the springs and strings and weights by which the forces were measured. The original statical experiments may also be regarded as the source of the principles of energy in connection with mechanical science. From this point of view the bodies upon which the forces act come into prominence, not because of their masses as in dynamics, but on account of their shapes, sizes and rigidity. Thus the experiments were made with levers, pulleys, inclined planes, wedges, etc.—in fact, with instruments for doing work, the mechanical powers of the text-books. In the statical principle of virtual velocities we have the origin of the principle of the equivalence of work and energy. To men of all times the most natural way of regarding force has been, as the action by which material is stretched, bent, twisted, broken or displaced, *i. e.*, whereby work is done. Even the word momentum, in the language of ordinary life, implies the power of doing work. It is worth consideration whether it may not be better in the instruction of students to work up to the ideas of dynamics through elementary examples of the equivalence of work and kinetic energy, rather than by taking the ordinary balloon passage to the laws of motion. While less systematic and formal, this procedure would be more natural and probably more useful.

The laws of energy may be summarized as follows. When work is done on a body an equivalent amount of energy is partly transformed and partly transferred without transformation. It is in general partly transmitted to other bodies with which the given body may be in physical connection. Its transformations are into stored energy and dissipated energy. Examples of stored energy are the potential energies due to gravitation, the forces of elasticity, magnetic and electrical attractions and

molecular forces. Such forces are termed conservative. Kinetic energy is another form of stored energy. Energy is dissipated by means of the forces of viscosity and friction, known as dissipative forces. Energy is also stored and dissipated in certain electrical, electro-magnetic, thermal, chemical and other actions which have not been identified with force and which, therefore, are not dynamical.

In order that work may be done there must be a source of energy, or place from which it comes, and a sink, or place to which it goes, together with an energy stream from the source to the sink. When work is done continuously the energy stream is accompanied by a circuit or system of stored energy which acts automatically as a moderator of its fluctuations.

The principle of conservation affirms that energy can neither be created nor destroyed, so that its changes are changes in form but not in amount.

The principle of the equivalence of work and energy is analogous to the second law of motion, considered as expressing the equivalence of impulse and momentum; that of the conservation of energy has its analogue in the third and first laws of motion regarded as affirming the conservation of momentum.

Newton notices this analogy in his scholium to the laws of motion in the words, "just as bodies in cases of collision have the same effect, whose velocities are inversely as their masses, so in putting machines in motion agents have the same effect, whose velocities in the directions of their forces are inversely as these forces." The now well known reference in the same scholium to the action of machines, the importance of which was pointed out in Thomson and Tait's Natural Philosophy, was in continuation of the same line of thought.

The impulse or time integral of a force is fully accounted for by the change of momentum, while the work or displacement integral is only partially accounted for by the change in kinetic energy, in all cases of real bodies. The reason for the difference is that the laws of motion are a complete statement of our experience of force in relation to the motion of a body as a whole, *i. e.*, the motion of its center of mass. On the other hand, the laws of energy require the consideration not only of this motion, but also of all internal motions and forces.

The principle of the equivalence of work and energy is a statement of an effect of force essentially different from its effect in producing change of momentum. It might be supposed, therefore, that this principle would be useful in affording another means of measuring force. The impossibility, in general, of measuring the whole change of energy due to an unknown force acting through an observed distance renders this idea to a great extent fruitless. If the laws of energy are true such a method of measuring force must give the same result as the dynamical method. The measurement of force by springs is based on this principle, and not on the second law of motion. Although no attempt is made to measure the change of energy due to the work of extending a spring, yet experience goes to show that the energy changes due to given extensions made in the same order are constant, and hence the corresponding forces are constant.

The connection between the laws of energy and those of motion may be stated as follows: Energy and work, like force, are fundamental conceptions gained from experience and having various relations with phenomena which can be discovered only as a result of experiment and observation. One of these relations is that work is proportional to the product of force into displacement. This relation is, therefore, a natural law, of the same order of impor-

tance as the second law of motion, and not a mere verbal definition. Experience thus gives a dynamical measure of work as well as of force.

The law of equivalence of work and energy then establishes work as a dynamical measure of energy.

The laws of motion, combined with this law of energy, establish the result that kinetic energy is proportional to the product of momentum and velocity, and thus furnish a dynamical method of measuring energy in its kinetic form. This is the sole contribution of the laws of motion to the science of energy.

It is only in the case of bodies whose internal forces and motions are known, or determinable from assumed data, in other words, imaginary bodies, that the laws of energy, as far as they are considered in dynamics, are included in the laws of motion and therefore become unnecessary, except for the purpose of convenience in mathematical analysis, or economy of thought. Even in such cases the expressions for work and energy retain a flavor of their original meaning and do not altogether degenerate into mere mathematical symbols.

The science of dynamics, as it is understood at the present day, includes among its fundamental principles, in addition to laws of motion, the principle of the equivalence of work and energy, and the principle of the conservation of energy; energy being measured, however, only in terms of force and displacement, or momentum and velocity.

The only actions known in dynamics are force and its integrals, impulse and work. To identify with these all other actions involving the transfer and transformation of energy, such as the conduction of heat, chemical reactions, induction of electric currents, etc., forms to-day the severest task of mathematical physics.

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MATHEMATICS AND PHYSICS AT THE BRITISH ASSOCIATION.

MEETING this year at Toronto in the week immediately succeeding the meeting of the American Association at Detroit, the British Association had the advantage of securing the attendance of a number of distinguished American scientists, who added greatly to the strength and interest of the proceedings. Taking Section A alone, it is sufficient to mention the names of Dr. Hill, Professors Michelson and Newcomb, as Vice-Presidents; and of Professors Barker, Carl Barus, Bedell, Carhart, Merritt, Nichols, Rosa, and many others who attended the meetings and assisted at the discussions in the work of the Section.

It is generally conceded, even by the rival sections, that A is not only the first but also the most strongly represented section, if not always in the number of its rank and file, at least in the distinction of its leaders and in the vigor and extent of its work. This year, in spite of the distance from home, formed no exception to the rule. Although some familiar faces were absent, the section formed a very strong and representative gathering of mathematicians and physicists. There were no less than fifty names on the committee, but it would have been easy to add to this number without going beyond the list of those attending the meeting whose work was already known.

In many ways, the extremely varied interests which the Section A represents are doubtless a great element of strength, but there are certain drawbacks in its excessive vitality. It brings together men from a number of different but closely allied departments of knowledge, who, if they did not possess some such common meeting ground, would be less able to keep in touch with each other, and to assist in the general advancement of science. At the same time it cannot be denied that the section is somewhat overburdened with an excess of com-